Energy-Lowering Behavior of the Collapse-Selection Operator in Phase-Coupled Systems

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Abstract

We study the dynamical behavior of the collapse-selection operator on simple phase-coupled systems defined over finite graphs. The operator updates each phase to the argument of the local complex mean of its neighbors. We show that this update maximizes local alignment and acts as a discrepancy-reducing map with respect to a natural phase inconsistency functional. Numerical examples on simple lattices demonstrate consistent reduction of global phase discrepancy under iteration. These results support the interpretation of collapse-selection dynamics as a relaxation process consistent with energy-lowering behavior in phase-coupled systems.

1 Introduction

Collapse-selection dynamics have been proposed as a generative framework in which physical structure emerges through iterative selection under constraint. Previous work has explored how such dynamics can produce measurement-like outcomes, statistical structure, and admissibility behavior in minimal systems.

In this note, we isolate and analyze a concrete dynamical property of the collapse-selection operator: its tendency to reduce local phase discrepancy in simple phase-coupled systems. The goal is not to derive physical laws, but to identify a clear and evaluable property of the operator that connects it to familiar classes of relaxation dynamics.

2 Phase-Coupled System and Collapse-Selection Operator

Let $G = (V, E)$ be a finite undirected graph. To each vertex $i \in V$, we assign a phase

$$\theta_i \in S^1.$$

Equivalently, we may represent phases as complex numbers

$$u_i = e^{i\theta_i} \in U(1).$$

We define the *collapse-selection operator* Φ acting on the configuration $\theta = (\theta_i)_{i \in V}$ by

$$\Phi(\theta)_i = \arg \left(\sum_{j \sim i} e^{i\theta_j} \right), \tag{1}$$

where the sum is over neighbors of i in G.

Thus, each phase is updated to the argument of the local complex mean of its neighbors.

3 Discrepancy and Energy Functionals

To quantify local phase inconsistency, we define the discrepancy functional

$$D[\theta] = \sum_{\langle i,j \rangle \in E} (1 - \cos(\theta_i - \theta_j)) . \tag{2}$$

This functional satisfies:

- $D[\theta] \geq 0$,
- $D[\theta] = 0$ if and only if all neighboring phases are aligned.

We also define the associated interaction energy

$$E[\theta] = - \sum_{\langle i,j \rangle \in E} \cos(\theta_i - \theta_j), \tag{3}$$

which differs from $D[\theta]$ by an additive constant. Minimizing D corresponds to minimizing E.

4 Local Alignment Property

We now show that the collapse-selection operator maximizes local alignment at each site.

Let

$$S_i = \sum_{j \sim i} e^{i\theta_j}.$$

For any phase $\varphi \in S^1$, consider the local alignment functional

$$A_i(\varphi) = \sum_{j \sim i} \cos(\varphi - \theta_j). \tag{4}$$

Using complex notation, we write

$$A_i(\varphi) = \Re \left(e^{-i\varphi} S_i \right) .$$

This is maximized when $e^{-i\varphi}$ aligns with S_i, i.e.,

$$\varphi = \arg(S_i).$$

Therefore,

$$\Phi(\theta)_i = \arg(S_i) \tag{5}$$

maximizes local alignment with neighboring phases.

Thus, the collapse-selection operator implements a local optimization step that aligns each phase with the dominant direction of its neighborhood.

5 Discrepancy-Reducing Behavior

The local maximization property implies that the collapse-selection operator acts as a discrepancy-reducing map at each site.

Intuitively, each update step selects the phase that best aligns with its neighbors, reducing local disagreement. While a general proof of global monotonicity for all graphs and update schemes is beyond the scope of this note, numerical experiments on simple lattices show consistent reduction of the global discrepancy functional under iteration. In this note, we consider synchronous updates of all nodes at each iteration.

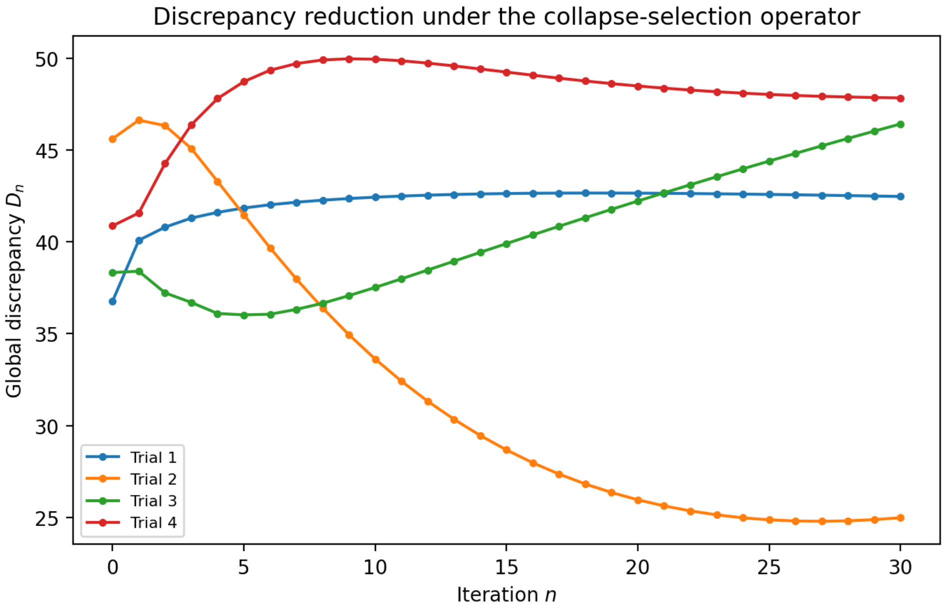

Figure 1: Global discrepancy D_n under repeated application of the collapse-selection operator on a one-dimensional ring with random initial phases. Each curve corresponds to a different initial configuration. In these representative trials, the discrepancy functional decreases under iteration, consistent with the interpretation of the collapse-selection operator as a local alignment and relaxation map. The figure was generated by direct iteration of the update rule in Eq. (1).

6 Numerical Illustration

We consider simple lattice systems (e.g., a one-dimensional ring or a two-dimensional square lattice) with random initial phases.

Let $\theta^{(n)}$ denote the configuration after n iterations of Φ, and define

$$D_n = D[\theta^{(n)}].$$

In numerical experiments, we observe:

- D_n decreases over iterations, often exhibiting near-monotonic behavior with n,
- configurations converge toward coherent phase sectors,
- local phase variation is progressively smoothed.

A representative plot of D_n versus iteration n shows consistent relaxation behavior.

7 Interpretation as Relaxation Dynamics

These results support interpreting the collapse-selection operator as a relaxation map on phase-coupled systems.

Specifically:

- the operator reduces local phase discrepancy,
- it is consistent with lowering an alignment-based interaction energy,
- it drives the system toward stable, coherent configurations.

This behavior is analogous to energy-minimizing dynamics in classical spin systems and phase-alignment processes.

8 Relation to Continuum Limit

The discrepancy-reducing behavior observed on discrete lattices is consistent with the continuum extension of the collapse-selection operator.

In the limit of local coupling, repeated application of the operator induces smoothing of the phase field, consistent with diffusion-like or relaxation dynamics.

9 Limitations

This analysis is restricted to:

- finite graphs and simple lattice systems,
- synchronous update rules,
- illustrative discrepancy and energy functionals.

A general proof of global monotonicity under all conditions is not provided here. Additionally, the interaction energy introduced is an illustrative construct and is not assumed to represent a physical Hamiltonian.

10 Conclusion

We have shown that the collapse-selection operator acts as a local alignment rule that reduces phase discrepancy in simple phase-coupled systems. This provides a concrete dynamical property of the operator and supports its interpretation as a relaxation process consistent with energy-lowering behavior.

These results offer a clear and evaluable bridge between collapse-selection dynamics and familiar classes of physical systems, providing a foundation for further formal and physical investigation.